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Errors in Orbital Predictions for Meteorological and Geodetic Satellites

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Abstract. The results of a theory which accounts for observed errors in orbital predictions are presented. The errors in predictions are assumed to arise from three causes: a sinusoidal variation in atmospheric density with a 27-day period, a random fluctuation in atmospheric density, and errors of observation. The theoretical errors are evaluated for Vanguard 1 and for the Tiros weather satellites as a function of the length of the predictions. Errors in predictions for the Anna 1-B geodetic satellite are expected to be approximately the same as for the Tiros satellites because the altitudes and ballistic parameters are similar. The theoretical errors are compared with errors in actual predictions issued by the Vanguard and National Aeronautics and Space Administration computing centers. The errors for Vanguard 1 are also given as a function of the year in which the prediction was made, illustrating the influence of perigee insolation. The problem of determining the location of clouds which appear on the Tiros weather photographs is discussed.

The atmosphere traversed by artificial satellites fluctuates in density in response to solar activity. This fact was discovered by Priester soon after the launching of Sputnik 1 [*Priester and Martin,* 1960] and has been confirmed and amplified by many other workers [*Jacchia,* 1961; *Paetzold,* 1962]. The fluctuations in atmospheric density are one cause of errors in orbital predictions [*Moe,* 1961; *Karrenberg et al.,* 1962], and they also introduce an ambiguity into the determination of definitive orbits [*Kaula,* 1963; *Moe,* 1963]. Knowledge of these density fluctuations has been used in constructing a theoretical model [*Moe,* 1960, 1962] of errors in orbital predictions. Reported in this paper are the errors in orbital predictions for satellites having the orbital characteristics and ballistic parameters typical of meteorological and geodetic satellites.

The variations in atmospheric drag employed in the model for errors in orbital predictions were derived from the orbital accelerations of the early satellites (Sputnik 3 and Vanguard 1). Data on the accelerations of satellites have continued to accumulate; some of the best data have been derived by *Jacchia and Slowey* [1962] from the observations of the balloon satellite Explorer 9. Their data are shown in Figure 1*a*, in which the upper curve shows the rates of change of period caused by atmospheric drag and the lower curve shows the rates of change of period caused by solar radiation pressure. The autocorrelation function [*Lawson and Uhlenbeck,* 1947] of the drag fluctuations is shown in Figure 1*b*. A periodicity of approximately 27 days, which is correlated with the motion of active regions across the solar disk, is evident in the autocorrelation function. The 'short-term autocorrelation function' in Figure 1*c* was obtained by removing the 27-day periodicity and trend from the orbital acceleration and computing the autocorrelation function of the drag fluctuations which remained. These short-term fluctuations have been shown to be correlated with geomagnetic activity [*Jacchia and Slowey,* 1962]. The indicated correlation time of 1 or 2 days is an upper bound because correlations are introduced by the procedure for deriving the orbital accelerations.

Orbital predictions are usually made by smoothing observations to determine the orbital elements and rate of change of period and then projecting these quantities ahead to predict a future time of equatorial crossing, using the assumption that the rate of change of period is constant. In the model [*Moe,* 1960, 1962], for the errors in orbital predictions it is assumed that the predicted time of equatorial crossing will be in error for three reasons: (1) the satellite acceleration is not constant, but varies with an approximately 27-day periodicity, (2) the acceleration also has a short-term random variation, and (3) errors of measurement are introduced by the tracking system. (Figures 1*b* and 1*c* illustrate the two components of drag variation.) A simple smoothing procedure was assumed in order to reduce the complexity of the statistical analysis. The three effects were inserted in the mathematical description of the smoothing procedure to obtain the position, period, and orbital acceleration referred to the center of the smoothing interval. These quantities were then used to 'predict' the time of the Nth equatorial crossing. The difference between the 'predicted' time and the actual time of the Nth equatorial crossing is the error. The following expressions were derived for the rms error caused by each of the three sources of error. The rms error, $S(N)$, caused by the sinusoidal variation in atmospheric density is given by

$$S(N) = (A/k^2)[(\alpha^2 + \beta^2)/2]^{1/2} \qquad (1)$$

where

$$\alpha = \cos(kN) - (2/ik)\sin(ik/2) + (64/i^3k)[N^2 - i(i+2)/12] \cdot [1 - \cos(ik/4)]\sin(ik/4)$$

$$\beta = \sin(kN) - kN + [8N/ik(i+2)] \cdot [\cos(ki/2) - 1 + i^2k^2/8]$$

and

$$A = 5.2h_p \mid D \mid \times 10^{-4}$$

where h_p is the height of perigee in kilometers, D is the smoothed rate of change of period (in minutes per revolution), i is the number of revolutions over which observations were smoothed to derive the orbital elements and rate of change of period, $k = 2\pi\ P/27$ (where P is the period in days), and N is the duration of the predictions measured in revolutions from the center of the smoothing interval.

The rms error, $R(N)$, caused by random drag fluctuations is

$$R(N) = F\{(N^3/3) + 2(i/4)^3 \cdot [(64/5)(N/i)^4 - 16(N/i)^3 + (N/i)^2]\}^{1/2} \qquad (2)$$

for

$$N \geq i/2 >> 1$$

where $F = 3.1h_p\,|D| \times 10^{-3}$. Within the smoothing interval ($N < i/2$), the expression for the random error is more complicated, but it can be calculated by the methods used in Appendixes D and G of *Moe* [1960].

The rms error, $O(N)$, caused by errors in the tracking observations, is

$$O(N) = (\sigma_o/i^2\sqrt{M})\left\{\begin{aligned}&256N^4 + 32Ni[i^2/3M - 4N^2/(M+2)]\\ &\quad + 16N^2i^2[M/(M+2) - (8/3)(M+2)/M - 2M/(M+2)^2]\\ &\quad + i^4[M/(M+2) + (16/9)(M+2)^2/M^2]\end{aligned}\right\}^{1/2} \qquad (3)$$

where M is the number of independent observations in the smoothing interval of i revolutions and σ_o is the equivalent observational error in minutes of time. The equivalent observational error of a 'semismooth' Minitrack observation is about 0.008 minute of time. There is approximately one semismooth Minitrack observation per revolution of the satellite.

On the assumption that the three errors are mutually independent, the total rms error in an orbital prediction is

$$E(N) = \{[R(N)]^2 + [S(N)]^2 + [O(N)]^2\}^{1/2} \qquad (4)$$

The errors in orbital predictions for Vanguard 1 near the time of sunspot maximum have been computed from equations 1, 2, 3, and 4. They are graphed in Figure 2 and compared with the rms error of twenty predictions issued by the Vanguard computing center in the autumn of 1958. The smoothing interval was 89 revolutions. Notice that the errors did not change greatly within the smoothing interval ($N < i/2 = 45$ revolutions) but increased rapidly outside.

Figure 3 shows a different kind of graph in which the error at the end of a 1- or 2-week prediction for Vanguard 1, issued by the Vanguard and National Aeronautics and Space Administration computing centers, is plotted as a

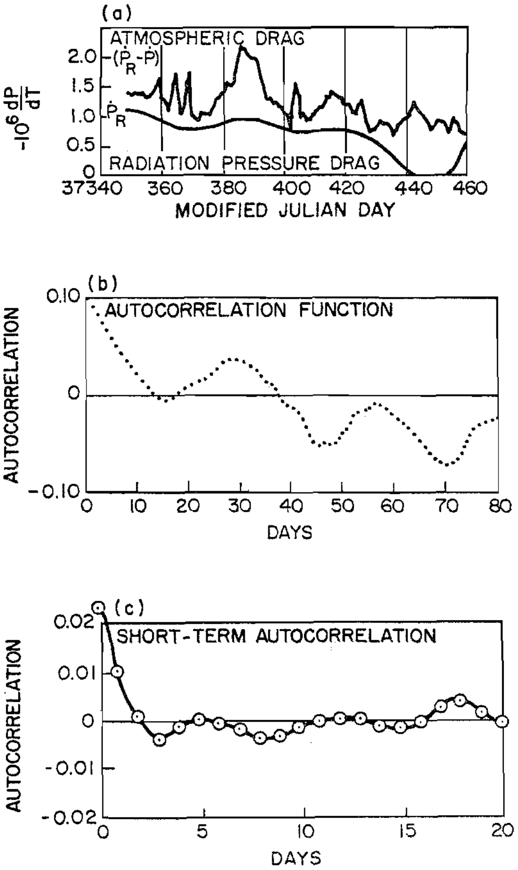

Fig. 1. Atmospheric drag and autocorrelation functions for Explorer 9.

function of the year in which the prediction was made. Because of oblateness perturbations, and the motion of the earth in its orbit, the position of perigee of Vanguard 1 moves in and out of the sunlight with a period of several years. The air drag is much larger in the sunlit hemisphere than in the dark hemisphere [*Priester and Martin,* 1960; *Jacchia,* 1961; *Paetzold,* 1962]. Since the fluctuations in atmospheric density (at a fixed altitude) are assumed in the model to be proportional to the atmospheric density itself, the errors should vary with a period of several years. Superimposed on this periodic variation is a slower downward trend due to the decrease in air density correlated with the waning of the sunspot cycle. It can be seen from the graph that as perigee passed from sunlight to twilight and into darkness the errors decreased, and the errors increased again as perigee passed back into the sunlight. When the perigee was in sunlight in 1958, the errors in predictions for Vanguard 1 were caused mainly by drag fluctuations, but when it was in darkness early in 1960 the errors were caused mainly by observational errors of the Minitrack system. (The theoretical curve for observational error in Figure 3 was higher in 1961 than in 1958 because the duration of the predictions was increased.) However, the actual errors in sunlight in 1961 were approximately twice as large as the theoretical model gave. Three possible reasons for this unexpected behavior are: (1) The fluctuations in atmospheric density have not decreased with the sunspot cycle in proportion to the mean density, (2) the Minitrack receiving system has deteriorated, or (3) the Vanguard transmitter has deteriorated. To test the first possibility, the correlation of geomagnetic activity with day-to-day changes in atmospheric density reported by *Jacchia and Slowey* [1962] for the 12-foot balloon satellite, Explorer 9, have been statistically analyzed (Moe, unpublished). The results definitely show that density fluctuations are much smaller at times of low geomagnetic activity, eliminating explanation (1). To test the second possibility, the recent orbital predictions for Tiros 4, 5, and 6 were examined. These continue to have small errors (in agreement with the model), indicating that the Minitrack receiving system has not deteriorated. In connection with the third possibility, the power produced by the solar cells of Vanguard 1 has decreased to approximately half of its initial value. This suggests the possibility that a reduction in transmitter power, resulting in a lower signal-to-noise ratio, is the cause of the increase of errors in orbital predictions for Vanguard 1 since early 1960. To obtain further information the errors in the predictions issued by the National Aeronautics and Space Administration computing center for Vanguard 1 were calculated for the fall of 1962, when perigee was in darkness, and the spring of 1963, when perigee had just passed into the sunlight. The rms error of prediction in each case was approximately 0.20 minute. The near constancy of the errors during the period 1961 to 1963 contrasts markedly with the large changes which occurred from 1958 to 1960, and again an increased observational error seems to be the cause.

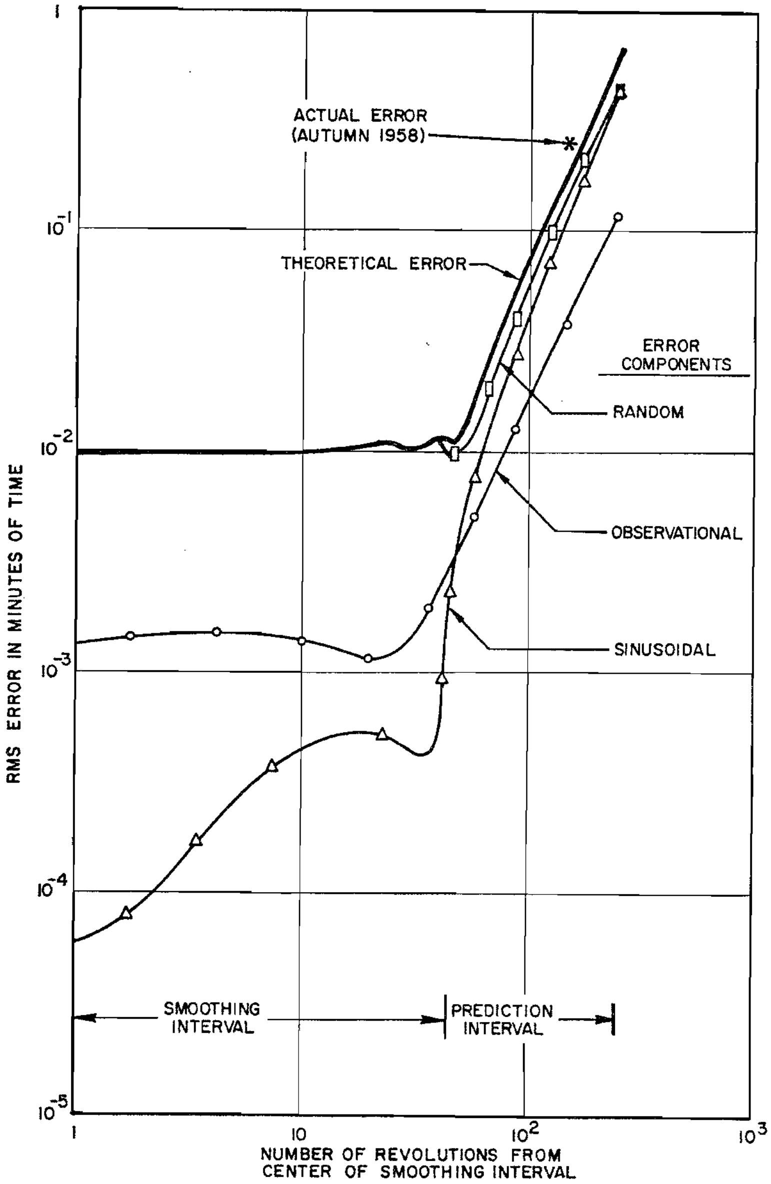

Fig. 2. Error in Vanguard 1 predictions near sunspot maximum.

The errors for Vanguard 1 with its perigee in darkness in 1960 are typical of the errors in orbital predictions for many recent satellites. These satellites have been placed in such high orbits that they are little affected by drag variations (balloon satellites have large errors of prediction, in spite of their high altitudes, because of their small ballistic parameters). The accuracy and abundance of Minitrack observations are the limiting factors which determine the errors for these satellites. The Tiros weather satellites and the Anna 1-B geodetic satellite have orbits of this type. Predictions for these satellites based on field-reduced Baker-Nunn camera observations would have approximately the same accuracy. Predictions based on the Applied Physics Laboratory Doppler system would have somewhat smaller errors (assuming

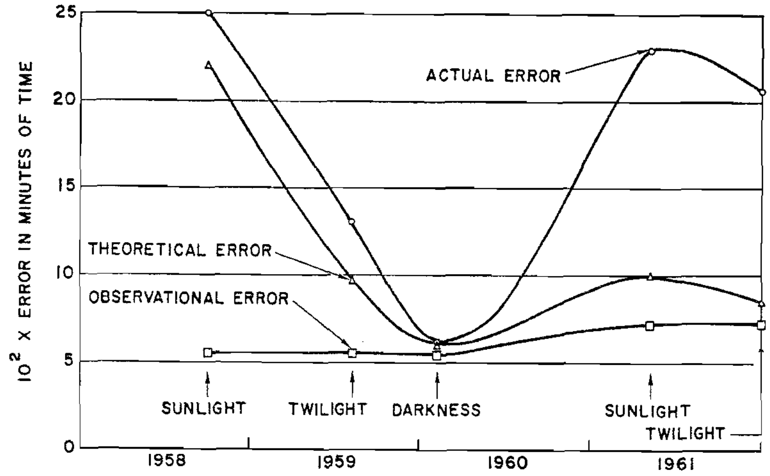

Fig. 3. Error in 1- to 2-week orbital predictions for Vanguard 1.

that the same smoothing and prediction intervals are used). Although predictions have no part in geodetic analysis, they are important in planning observations by optical instruments having small fields of view, e.g., ballistic cameras and astronomical telescopes.

Figure 4 shows the errors in the 1- to 2-week orbital predictions tangential to the path of the Tiros weather satellites as a function of the number of revolutions from the center of the smoothing interval. The Tiros 1, 2, and 3 satellites had very similar orbits. The theoretical

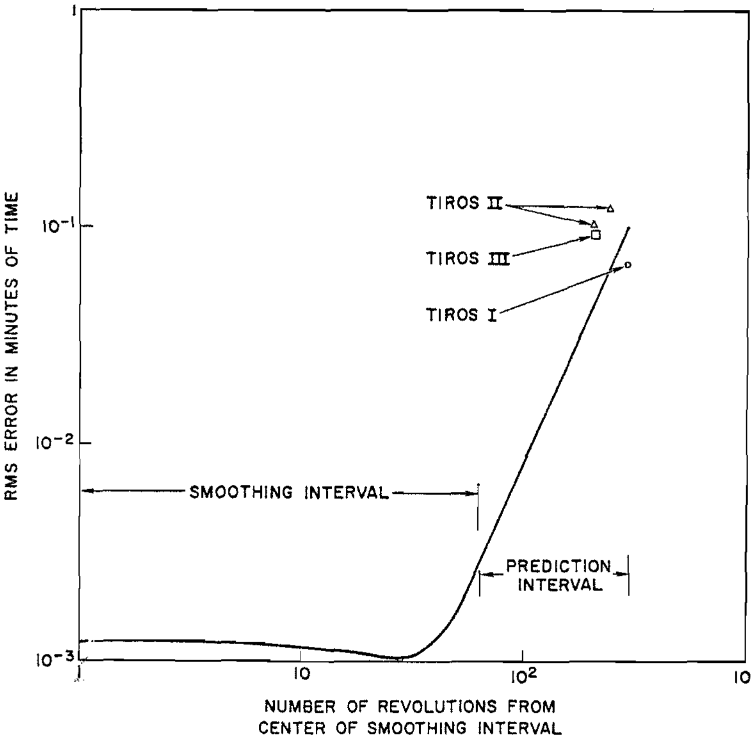

Fig. 4. Error of orbital predictions for Tiros weather satellites.

curve for only one of them is shown because the theoretical curves differ less than the actual errors do. The Tiros satellites were so high that the errors in predictions for them were caused almost entirely by the observational errors of the Minitrack stations. The rms errors of groups of actual predictions for the three Tiros satellites are indicated by the circle, triangles, and square in Figure 4. An error of 0.1 minute of time is equivalent to a positional error of approximately 40 km tangential to the path. The errors at right angles to the path are an order of magnitude smaller.

Some of the meteorologists who work with the Tiros weather pictures would like to use the predictions to locate points on the pictures with better accuracy. When points on the ground can be recognized in the photographs, the clouds can be located by photogrammetric means with better accuracy than they can from the predicted orbital position. But when the surface features are unrecognizable, the clouds must be located by combining the orbital position with the orientation of the satellite axes. The standard deviation of the orbital position varies from ½ to 2 km along the fitted orbit (i.e., within the smoothing interval) and from 2 to 40 km along the predicted orbit. Therefore, if a fitted orbit is available, there will be a smaller error in satellite position than if a prediction is used.

When ground points are unrecognizable, there are three sources of attitude information: infrared sensors placed at 90° to the spin axis, infrared sensors placed at 45° to the spin axis, and a mathematical model in which the magnetic moment, gravity gradient, and eddy currents are used. The mathematical model can be used along with other attitude data (J. V. Natrella, unpublished discussion, 1962). An examination of the Tiros 3 attitude during July and August 1961 computed by Natrella indicated that the standard deviations of the right ascension and declination of the spin axis were 2° and 3°, respectively. At a slant range of 1000 km, an error of 3° causes a positional error of 47 km. Although the error in orbital position can be reduced to 2 km by using the fitted orbit rather than a prediction, the attitude determination can still produce an error of approximately 50 km, which would be quite difficult to reduce. The best hope of locating clouds with better accuracy, in the opinion of the writer, lies in exploiting the method of variable development of the Tiros photographs, as described, for example, by *Mendoza and Vasques* [1962].

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